- Egor Kosov

  "Sampling discretization problem for L^1 norm"

Abstract

In the talk we discuss new bounds for the number of points sufficient for good discretization of L^1 norm on finite dimensional subspaces, for which a certain Nikolskii-type inequality is valid.

- Alexei Shadrin (University of Cambridge, UK)

  "On cardinality of the lower sets and their universal discretization"

Abstract

Recently, the algebraic and trigonometric polynomials whose powers form the so-called lower (or downward closed) sets became a noticeable topic of studies in approximation theory. In this talk, we point out its not very well known connection to some problems in number theory, statistical mechanics, combinatorics and functional analysis. We provide new upper and lower bounds for cardinality of the lower sets, and use these bounds to derive Marcinkievicz-type results on universal discretisation of the collection of all n-dimensional subspaces of polynomials formed with respect to such sets.

- Jan Vybíral (Czech Technical University Praha, Czech Republic)

  "Robust and efficient identification of neural networks"

Abstract

Abstract: We address the structure identification and the uniform approximation of sums of ridge functions $f(x) = \sum_{i=1}^{m} g_i(\langle a_i, x\rangle)$ on $\mathbb{R}^d$, representing a general form of a shallow feed-forward neural network, from a small number of query samples. Higher order differentiation, as used in our constructive approximations, of sums of ridge functions or of their compositions, as in deeper neural network, yields a natural connection between neural network weight identification and tensor product decomposition identification. In the case of the shallowest feed-forward neural network, second order differentiation and tensors of order two (i.e., matrices) suffice. Based on multiple approximated first and second order differentials, our general approximation strategy is developed as a sequence of algorithms to perform individual sub-tasks. We first perform an active subspace search by approximating the span of the weight vectors $a_1, \ldots, a_m$. Then we use a

straightforward substitution, which reduces the dimensionality of the problem from d to m. The core of the construction is then the stable and efficient approximation of weights expressed in terms of rank-1 matrices $a_i \otimes a_i$, realized by formulating their individual identification as a suitable nonlinear program. We prove the successful identification by this program of weight vectors being close to orthonormal and we also show how we can constructively reduce to this case by a whitening procedure, without loss of any generality. We finally discuss the implementation and the performance of the proposed algorithmic pipeline with extensive numerical experiments, which illustrate and confirm the theoretical results.

- Felix Bartel (Chemnitz University of Technology, Germany)

  "Cross-validation in Scattered Data Approximation"

Abstract

Cross-validation is a classical tool in the learner's repertoire to compare the goodness of fit for different reconstruction models. This utilizes only the function values but, on the downside, comes with a high computational cost if implemented naively.

In this talk we present an approach to shift the main computations from the function in question to the node distribution in the case of penalized least squares estimation (PLSE). Given exact quadrature, this is further simplified such that the computation of the cross-validation score is computationally as expensive as computing the PLSE itself.

Furthermore, we present results on the theoretical properties of the cross-validation score. So far, most optimality results are stated in an asymptotic fashion. We propose a concentration inequality on the difference of cross-validation score and L2-error. This gives a pre-asymptotic bound which holds with high probability. For the assumptions we rely on bounds on the uniform error of the model which allow for a broadly applicable framework. We support our claims by applying this machinery to Shepard's model, where we are able to determine precise constants of the concentration inequality.

- Giovanni Migliorati (Laboratoire Jacques-Louis Lions, Sorbonne Université, France)

  "Adaptive approximation by weighted least squares"

- Kateryna Pozharska (Institute of Mathematics of NAS of Ukraine)

  "Sampling recovery of multivariate functions in the uniform norm"

Abstract

The talk is about the recovery of multivariate functions from reproducing kernel Hilbert spaces in the uniform norm. Surprisingly, a certain weighted least squares recovery operator which uses random samples from a tailored distribution leads to near optimal results in several relevant situations. The results are stated in terms of the decay of related singular numbers of the compact embedding into $L_2(D)$ multiplied with the supremum of the Christoffel function of the subspace spanned by the first m singular functions. As an application we discuss new recovery guarantees for Sobolev type spaces related to Jacobi type differential operators on the one hand and classical multivariate periodic Sobolev type spaces with general smoothness weight on the other hand. By applying a recently introduced sub-sampling technique related to Weaver's conjecture we mostly lose a $\sqrt{\log n}$ factor compared to the optimal worst-case error and sometimes even less.

- Mark Iwen (Michigan State University, USA)

  "Sparse Fourier Transforms on Rank-1 Lattices for the Rapid and Low-Memory Approximation of Functions of Many Variables, + Generalizations"

Abstract

We will discuss fast and provably accurate algorithms for approximating smooth functions on the d-dimensional torus which admit accurate s-sparse approximations in the Fourier basis for frequencies confined to ,e.g., a finite hyperbolic cross set H in Z^d. Both deterministic and explicit as well as randomized algorithms for solving this problem using $O(s^2 dlog^c(|H|))$-time/memory/samples and $O(sdlog^c(|H|)$-time/memory/samples, respectively. Most crucially, all of the methods proposed herein achieve these runtimes and sampling complexities while simultaneously satisfying theoretical best s-term approximation guarantees which ensure their numerical accuracy and robustness to noise for general functions with absolutely convergent Fourier series. Generalizations of similar fast techniques to other orthonormal bases will also be briefly discussed, time permitting.

- Yu. Malykhin, K. Ryutin, T.Zaitseva

  "The recovery of ridge functions"

- Irina Limonova

  "On sampling discretization in L_2"

Abstract

The talk is based on the joint paper with Temlyakov (arXiv:2009.10789v2). We prove a sampling discretization theorem for the square norm of functions from a finite dimensional subspace satisfying Nikol'skii's inequality with an upper bound on the number of sampling points of the order of the dimension of the subspace.

- Glenn Byrenheid (Friedrich-Schiller University of Jena, Germany)

  "Constructive sparse approximation based on sampling"

Abstract

We consider the approximation of multivariate functions by suitable m-term truncations of their tensorized Faber-Schauder series expansions. The univariate Faber-Schauder system on $[0, 1]$ is given by dyadic dilates and translates (in the wavelet sense) of simple hat functions with support in $[0, 1]$. We obtain a hierarchical basis which will be tensorized over all levels (hyperbolic) to get the dictionary $\mathcal{F}$. The worst-case error with respect to a class of functions $\mathbb{F} \hookrightarrow X$ is measured by the usual best m-term widths denoted by $\sigma_m(\mathbb{F}, \mathcal{F})_X$, where the error is measured in X. We constructively prove the following sharp asymptotical bound for the class of Besov spaces with small mixed smoothness (i.e. $1/p < r < \min\{1/\theta - 1, 2\}$) in $L_q (p < q \leq 1)$

$$\sigma_m(S^r_{p,\theta}B, \mathcal{F})_q \asymp m^{-r}\,.$$

Note, this asymptotical rate of convergence does not depend on the dimension d (only the constants behind). In addition, this result holds for $q = \infty$ and to our best knowledge this is one of the very few known sharp results involving L_∞ as a target space for sparse approximation in the context of spaces with dominating mixed smoothness. We emphasize two more things. First, the selection procedure for the coefficients is a level-wise constructive greedy strategy which only touches a nite prescribed number of coefficients. And second, due to the use of the Faber-Schauder system, the coefficients are nite linear combinations of discrete function values. Hence, this method can be considered as a nonlinear adaptive sampling algorithm.

- Stefan Kunis (Technical University of Osnabrück, Germany)

  "A survey condition number estimates for multivariate nonequispaced Fourier matrices"

Abstract

L^2-variants of classical Marcinkiewicz-Zygmund and Ingham inequalities are equivalent to bounding the extremal singular values of nonequispaced Fourier matrices. The talk surveys some known results and focuses on multivariate analogues under simple geometric assumptions on the sampling nodes. Some applications for subspace methods in so-called superresolution imaging are presented as well.

- Michael Schmischke (Chemnitz University of Technology, Germany)

  "High-dimensional interpretable approximation of functions with low effective dimension"

Abstract

In this talk we present a method for the approximation of high-dimensional functions. We assume that the function consists mostly of low-order interactions, i.e., has a low superpositon dimension. In practical applications this is reffered to as sparsity-of-effects. The method is based upon the analysis of variance (ANOVA) decomposition. Using global sensitivity indices or Sobol indices we are able to interpret our model, i.e., rank the importance of variables and variable interactions. We present numerical results for synthetic and real data applications.

- Lutz Kämmerer (Chemnitz University of Technology, Germany)

  "A sample efficient sparse FFT for arbitrary frequency candidate sets in high dimensions"

Abstract

In this talk, an algorithm is presented for the efficient reconstruction of functions that can be represented by just few out of a potentially large and possibly unstructured candidate set of Fourier basis functions in high spatial dimensions, a so-called high-dimensional sparse fast Fourier transform. In contrast to many other such algorithms, our method works for arbitrary candidate sets and does not make additional structural assumptions on the candidate set. Our transform significantly improves upon the other approaches available for such

a general framework in terms of the scaling of the sample complexity. The algorithm is based on sampling the function along multiple rank-1 lattices with random generators. Combined with a dimension-incremental approach, our method yields a sparse Fourier transform whose computational complexity only grows mildly in the dimension and can hence be efficiently computed even in high dimensions. The theoretical analysis establishes that any Fourier s-sparse function can be accurately reconstructed with high probability.